EXAMPLE 3 Given $f(x) = \frac{1}{x}$ and $g(x) = 2x - 7$, find the domains of $f + g$, $f - g$, $f \cdot g$, and f/g.

SOLUTION We first find the domain of f and the domain of g:

Find the domains of f and g.

The domain of f is $\{x \mid x \text{ is a real number } and\ x \neq 0\}$.
The domain of g is $\mathbb{R}$.

The domains of $f + g$, $f - g$, and $f \cdot g$ are the set of all elements common to the domains of f and g. This consists of all real numbers except 0.

Find the domain of $f + g$, $f - g$, and $f \cdot g$.

$$\text{The domain of } f + g = \text{the domain of } f - g = \text{the domain of } f \cdot g = \{x \mid x \text{ is a real number } and\ x \neq 0\}.$$

Find any values of x for which $g(x) = 0$.

Because we cannot divide by 0, the domain of f/g must also exclude any values of x for which $g(x)$ is 0. We determine those values by solving $g(x) = 0$:

$$\begin{aligned} g(x) &= 0 \\ 2x - 7 &= 0 \qquad \text{Replacing } g(x) \text{ with } 2x - 7 \\ 2x &= 7 \\ x &= \tfrac{7}{2}. \end{aligned}$$

Find the domain of f/g.

The domain of f/g is the domain of the sum, the difference, and the product of f and g, found above, excluding $\frac{7}{2}$.

$$\text{The domain of } f/g = \{x \mid x \text{ is a real number } and\ x \neq 0\ and\ x \neq \tfrac{7}{2}\}.$$

Try Exercises 47 and 55.

Division by 0 is not the only condition that can force restrictions on the domain of a function. In Chapter 7, we will examine functions similar to that given by $f(x) = \sqrt{x}$, for which the concern is taking the square root of a negative number.

2.5 Exercise Set

Concept Reinforcement *Make each of the following sentences true by selecting the correct word for each blank.*

1. The function $f - g$ is the ___difference___ (sum/difference) of f and g.

2. One way to compute $(f - g)(2)$ is to ___subtract___ (erase/subtract) $g(2)$ from $f(2)$.

3. One way to compute $(f - g)(2)$ is to simplify $f(x) - g(x)$ and then ___evaluate___ (evaluate/substitute) the result for $x = 2$.

4. The domain of $f + g$ is the set of all values ___common to___ (common to/excluded from) the domains of f and g.

5. The domain of f/g is the set of all values common to the domains of f and g, ___excluding___ (including/excluding) any values for which $g(x)$ is 0.

6. The height of $(f + g)(a)$ on a graph is the ___sum___ (product/sum) of the heights of $f(a)$ and $g(a)$.

Let $f(x) = -3x + 1$ and $g(x) = x^2 + 2$. Find each of the following.

7. $f(2) + g(2)$ 1	8. $f(-1) + g(-1)$ 7
9. $f(5) - g(5)$ −41	10. $f(4) - g(4)$ −29
11. $f(-1) \cdot g(-1)$ 12	12. $f(-2) \cdot g(-2)$ 42

13. $f(-4)/g(-4)$ $\frac{13}{18}$ **14.** $f(3)/g(3)$ $-\frac{8}{11}$

15. $g(1) - f(1)$ 5 **16.** $g(2)/f(2)$ $-\frac{6}{5}$

17. $(f + g)(x)$ $x^2 - 3x + 3$ **18.** $(g - f)(x)$ $x^2 + 3x + 1$

19. $(f - g)(x)$ $-x^2 - 3x - 1$ **20.** $(g/f)(x)$ ⊡

Let $F(x) = x^2 - 2$ and $G(x) = 5 - x$. Find each of the following.

21. $(F + G)(x)$ $x^2 - x + 3$ **22.** $(F + G)(a)$ $a^2 - a + 3$

23. $(F + G)(-4)$ 23 **24.** $(F + G)(-5)$ 33

25. $(F - G)(3)$ 5 **26.** $(F - G)(2)$ -1

27. $(F \cdot G)(-3)$ 56 **28.** $(F \cdot G)(-4)$ 126

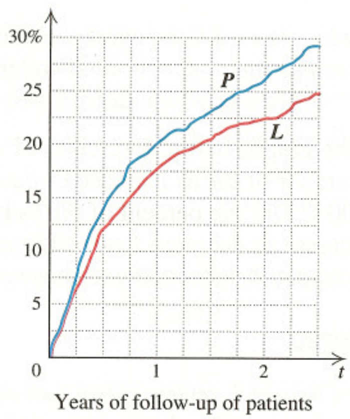
29. $(F - G)(a)$ $a^2 + a - 7$ **30.** $(F/G)(x)$ $\frac{x^2 - 2}{5 - x}, x \neq 5$

31. $(F - G)(x)$ $x^2 + x - 7$ **32.** $(G - F)(x)$ $-x^2 - x + 7$

33. $(F/G)(-2)$ $\frac{2}{7}$ **34.** $(F/G)(-1)$ $-\frac{1}{6}$

In 2004, a study comparing high doses of the cholesterol-lowering drugs Lipitor and Pravachol indicated that patients taking Lipitor were significantly less likely to have heart attacks or require angioplasty or surgery.

In the graph below, $L(t)$ is the percentage of patients on Lipitor (80 mg) and $P(t)$ is the percentage of patients on Pravachol (40 mg) who suffered heart problems or death t years after beginning to take the medication.

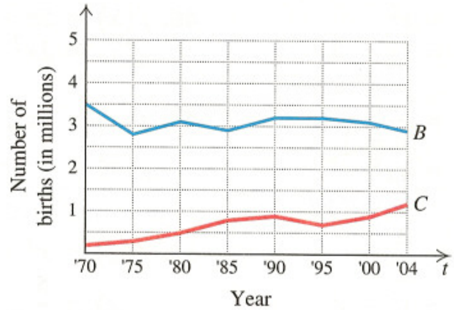


Source: New England Journal of Medicine

35. Use estimates of $P(2)$ and $L(2)$ to estimate $(P - L)(2)$. 4%

36. Use estimates of $P(1)$ and $L(1)$ to estimate $(P - L)(1)$. 2%

The graph below shows the number of births in the United States, in millions, from 1970–2004. Here $C(t)$ represents the number of Caesarean section births, $B(t)$ the number of non-Caesarean section births, and $N(t)$ the total number of births in year t.



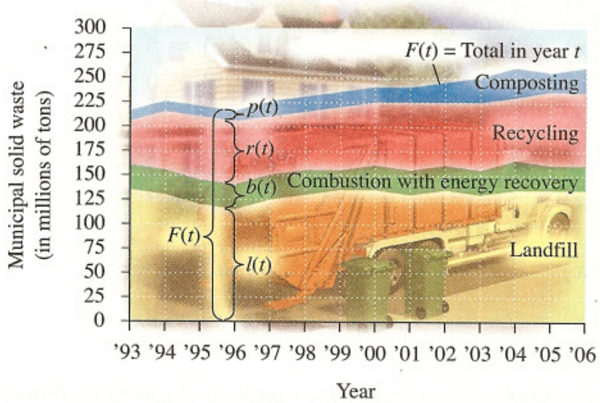
Source: National Center for Health Statistics

37. Use estimates of $C(2004)$ and $B(2004)$ to estimate $N(2004)$. $1.2 + 2.9 = 4.1$ million births

38. Use estimates of $C(1985)$ and $B(1985)$ to estimate $N(1985)$. $0.8 + 2.9 = 3.7$ million births

Often function addition is represented by stacking the individual functions directly on top of each other. The graph below indicates how U.S. municipal solid waste has been managed. The braces indicate the values of the individual functions.

Talking Trash



Source: Environmental Protection Agency

39. Estimate $(p + r)('05)$. What does it represent? ⊡

40. Estimate $(p + r + b)('05)$. What does it represent? ⊡

41. Estimate $F('96)$. What does it represent? ⊡

42. Estimate $F('06)$. What does it represent? ⊡

43. Estimate $(F - p)('04)$. What does it represent? ⊡

44. Estimate $(F - l)('03)$. What does it represent? ⊡

⊡ Answers to Exercises 20 and 39–44 are on p. IA-6.

For each pair of functions f and g, determine the domain of the sum, the difference, and the product of the two functions.

45. $f(x) = x^2$, $g(x) = 7x - 4$ $\mathbb{R}$

46. $f(x) = 5x - 1$, $g(x) = 2x^2$ $\mathbb{R}$

47. $f(x) = \dfrac{1}{x - 3}$, $g(x) = 4x^3$ $\{x \mid x \text{ is a real number } and\ x \neq 3\}$

48. $f(x) = 3x^2$, $g(x) = \dfrac{1}{x - 9}$ $\{x \mid x \text{ is a real number } and\ x \neq 9\}$

49. $f(x) = \dfrac{2}{x}$, $g(x) = x^2 - 4$ $\{x \mid x \text{ is a real number } and\ x \neq 0\}$

50. $f(x) = x^3 + 1$, $g(x) = \dfrac{5}{x}$ $\{x \mid x \text{ is a real number } and\ x \neq 0\}$

51. $f(x) = x + \dfrac{2}{x - 1}$, $g(x) = 3x^3$ $\{x \mid x \text{ is a real number } and\ x \neq 1\}$

52. $f(x) = 9 - x^2$, $g(x) = \dfrac{3}{x - 6} + 2x$ $\{x \mid x \text{ is a real number } and\ x \neq 6\}$

53. $f(x) = \dfrac{x}{2x - 9}$, $g(x) = \dfrac{5}{1 - x}$ ⊡

54. $f(x) = \dfrac{5}{3 - x}$, $g(x) = \dfrac{x}{4x - 1}$ ⊡

For each pair of functions f and g, determine the domain of f/g.

55. $f(x) = x^4$, $g(x) = x - 3$ $\{x \mid x \text{ is a real number } and\ x \neq 3\}$

56. $f(x) = 2x^3$, $g(x) = 5 - x$ ⊡

57. $f(x) = 3x - 2$, $g(x) = 2x - 8$ $\{x \mid x \text{ is a real number } and\ x \neq 4\}$

58. $f(x) = 5 + x$, $g(x) = 6 - 2x$ ⊡

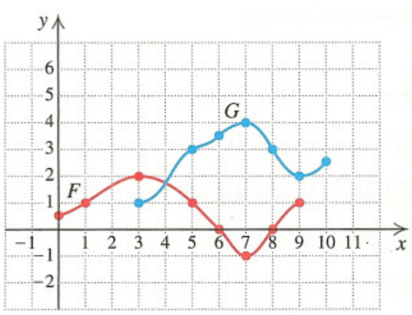
59. $f(x) = \dfrac{3}{x - 4}$, $g(x) = 5 - x$ ⊡

60. $f(x) = \dfrac{1}{2 - x}$, $g(x) = 7 - x$ ⊡

61. $f(x) = \dfrac{2x}{x + 1}$, $g(x) = 2x + 5$ ⊡

62. $f(x) = \dfrac{7x}{x - 2}$, $g(x) = 3x + 7$ ⊡

For Exercises 63–70, consider the functions F and G as shown.



63. Determine $(F + G)(5)$ and $(F + G)(7)$. 4; 3

64. Determine $(F \cdot G)(6)$ and $(F \cdot G)(9)$. 0; 2

65. Determine $(G - F)(7)$ and $(G - F)(3)$. 5; −1

66. Determine $(F/G)(3)$ and $(F/G)(7)$. 2; $-\frac{1}{4}$

67. Find the domains of $F, G, F + G$, and F/G. ⊡

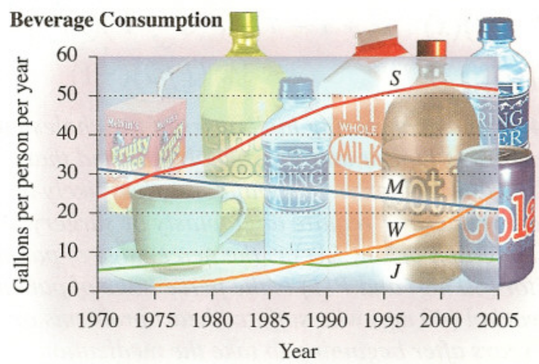
68. Find the domains of $F - G, F \cdot G$, and G/F. $\{x \mid 3 \leq x \leq 9\}$; $\{x \mid 3 \leq x \leq 9\}$; $\{x \mid 3 \leq x \leq 9$ *and* $x \neq 6$ *and* $x \neq 8\}$

69. Graph $F + G$. ⊡

70. Graph $G - F$. ⊡

⊡ Answers to Exercises 53, 54, 56, 58–62, 67, 69, and 70 are on p. IA-6.

In the graph below, $S(t)$ represents the number of gallons of carbonated soft drinks consumed by the average American in year t, $M(t)$ the number of gallons of milk, $J(t)$ the number of gallons of fruit juice, and $W(t)$ the number of gallons of bottled water.



Source: Economic Research Service, U. S. Department of Agriculture

TW **71.** Between what years did the average American drink more soft drinks than juice, bottled water, and milk combined? Explain how you determined this.

TW **72.** Examine the graphs before Exercises 37 and 38. Did the total number of births increase or decrease from 1970 to 2004? Did the percent of births by Caesarean section increase or decrease from 1970 to 2004? Explain how you determined your answers.

SKILL REVIEW

To prepare for Chapter 3, review solving an equation for y and translating phrases to algebraic expressions (Sections 1.6 and 1.7).

Solve. [1.6]

73. $x - 6y = 3$, for y $y = \frac{1}{6}x - \frac{1}{2}$

74. $3x - 8y = 5$, for y $y = \frac{3}{8}x - \frac{5}{8}$

75. $5x + 2y = -3$, for y $y = -\frac{5}{2}x - \frac{3}{2}$

76. $x + 8y = 4$, for y $y = -\frac{1}{8}x + \frac{1}{2}$

Translate each of the following. Do not solve. [1.7]

77. Five more than twice a number is 49. Let n represent the number; $2n + 5 = 49$

78. Three less than half of some number is 57. Let x represent the number; $\frac{1}{2}x - 3 = 57$

79. The sum of two consecutive integers is 145.
Let x represent the first integer; $x + (x + 1) = 145$

80. The difference between a number and its opposite is 20.
Let n represent the number; $n - (-n) = 20$

SYNTHESIS

TW **81.** Examine the graphs showing number of calories expended following Example 2 and explain how similar graphs could be drawn to represent the absorption of 200 mg of Advil® taken four times a day.

TW **82.** If $f(x) = c$, where c is some positive constant, describe how the graphs of $y = g(x)$ and $y = (f + g)(x)$ will differ.

83. Find the domain of f/g, if

$$f(x) = \frac{3x}{2x + 5} \quad \text{and} \quad g(x) = \frac{x^4 - 1}{3x + 9}.$$ ☐

84. Find the domain of F/G, if

$$F(x) = \frac{1}{x - 4} \quad \text{and} \quad G(x) = \frac{x^2 - 4}{x - 3}.$$

$\{x \mid x \text{ is a real number and } x \neq 4 \text{ and } x \neq 3 \text{ and } x \neq 2 \text{ and } x \neq -2\}$

85. Sketch the graph of two functions f and g such that the domain of f/g is

$$\{x \mid -2 \leq x \leq 3 \text{ and } x \neq 1\}.$$ ☐

86. Find the domains of $f + g$, $f - g$, $f \cdot g$, and f/g, if

$$f = \{(-2, 1), (-1, 2), (0, 3), (1, 4), (2, 5)\}$$

and

$$g = \{(-4, 4), (-3, 3), (-2, 4), (-1, 0), (0, 5), (1, 6)\}.$$ ☐

87. Find the domain of m/n, if

$$m(x) = 3x \text{ for } -1 < x < 5$$

and $\{x \mid x \text{ is a real number and } -1 < x < 5 \text{ and } x \neq \frac{3}{2}\}$

$$n(x) = 2x - 3.$$

88. For f and g as defined in Exercise 86, find $(f + g)(-2)$, $(f \cdot g)(0)$, and $(f/g)(1)$. $5; 15; \frac{2}{3}$

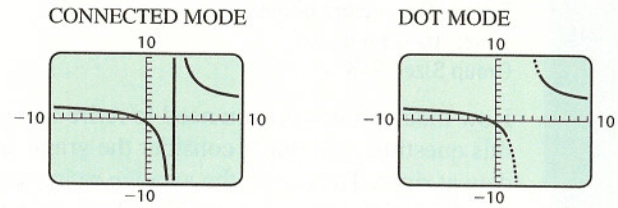
89. Write equations for two functions f and g such that the domain of $f + g$ is

$$\{x \mid x \text{ is a real number and } x \neq -2 \text{ and } x \neq 5\}.$$ ☐

90. Using the window $[-5, 5, -1, 9]$, graph $y_1 = 5$, $y_2 = x + 2$, and $y_3 = \sqrt{x}$. Then predict what shape the graphs of $y_1 + y_2$, $y_1 + y_3$, and $y_2 + y_3$ will take. Use a graph to check each prediction.

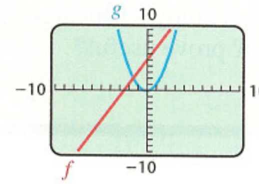
☐ Answers to Exercises 83, 85, 86, 89, and 91 are on p. IA-6.

91. Let $y_1 = 2.5x + 1.5$, $y_2 = x - 3$, and $y_3 = y_1/y_2$. For many calculators, depending on whether the CONNECTED or DOT mode is used, the graph of y_3 appears as follows.

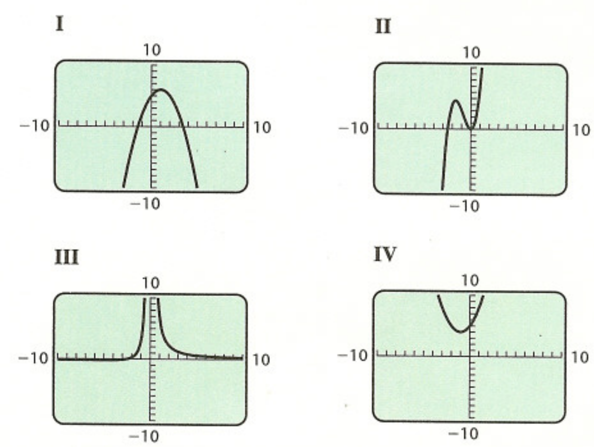


Use algebra to determine which graph more accurately represents y_3. ☐

92. Use the graphs of f and g, shown below, to match each of $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $(f/g)(x)$ with its graph.



a) $(f + g)(x)$ IV
b) $(f - g)(x)$ I
c) $(f \cdot g)(x)$ II
d) $(f/g)(x)$ III



Try Exercise Answers: Section 2.5

7. 1 **17.** $x^2 - 3x + 3$
47. $\{x \mid x \text{ is a real number and } x \neq 3\}$
55. $\{x \mid x \text{ is a real number and } x \neq 3\}$